UNPUBLISHED PRELIMINARY DATA

"The Helical Motion of a Sphere in the Presence of a Magnetic Field"

By

David P. Hoult

The Pennsylvania State University
Ionosphere Research Laboratory
University Park, Pennsylvania

Abstract

16269

When a sphere is released in a tank of mercury, in the presence of a uniform applied magnetic field aligned with the direction of gravity, the sphere is observed, for some values of the applied magnetic field, to rise in a steady helical motion. A theory is presented which describes this effect. The main results are relationships between the drag on the sphere, the wave length of the helix, and the strength of the applied magnetic field: these relations must be satisfied if a steady motion occurs. It is believed that this calculation is a new application of the theory of hydrodynamic stability of parallel flow. Author

I. Introduction

For viscous Reynolds numbers, Re ($= U D/2\nu$, U = vertical velocity of the sphere, D = sphere diameter, ν = kinematic viscosity), greater than 10^4, a sphere rising (or falling) in an incompressible fluid, does not rise in a straight line as intuition based on symmetry would suggest. Rather the motion is oscillatory. The deviations from a straight path are smaller the larger the ratio of sphere density to fluid density. It has been observed[1] that a rather large magnetic field stabilizes these oscillations, and that the sphere rises in a steady helical path. This paper presents a theory which describes this motion.

The use of a small disturbance theory to describe this motion is justified provided the steady vertical velocity of the sphere is much greater than the horizontal velocity, V, ($V/U \ll 1$). The results of parallel flow stability calculations[2] (reference (2) is hereafter denoted (I)) will be used. This is appropriate because, if the magnetic field is sufficiently large, the wake of the sphere[3,4,5] extends to upstream infinity. The sphere is located in the center of a wake extending from upstream to downstream infinity. The stability calculation ignores viscosity; it is known that[6] the unstable mode of an axisymmetric wake is little changed by viscous effects provided the Reynolds number is large. It is assumed that the unstable mode is unchanged by the presence of the sphere. Near the sphere, this is certainly wrong. But the helical motion has a characteristic size of many sphere diameters, and it seems reasonable to describe this large scale motion without taking into account the details of the flow near the sphere.

The basic physical assumption is the following: in the presence of a large magnetic field, the wake of a free rising sphere will be nearly aligned with the direction of the magnetic field. This means that the axial wave number, α, of the disturbance is very small ($\alpha = \pi D/\lambda$; the disturbance varies in the vertical direction as $\exp(i\, 2\pi z/\lambda)$), i.e., the wake is nearly straight. The form of the disturbance is taken to be the limit, as $\alpha \to 0$, of the 'helical' unstable mode of an axisymmetric wake. The sphere moves with the local horizontal velocity of the disturbance. The force due to the disturbance pressure on the sphere is composed of two parts; a thrust balanced by the drag component due to the transverse velocity, and a lift balanced by the radial acceleration of the sphere.

Section II describes the steady flow over a sphere in the presence of a large magnetic field. Section III gives the relevant results of small disturbance theory as presented in (I). Section IV presents the calculation describing the helical motion of the sphere.

II. The Steady Motion

Consider a sphere rising vertically in an incompressible conducting fluid due to the action of buoyancy. The magnetic Reynolds number of the motion, Rm, ($= \mu\sigma U D/2$, μ = magnetic permeability, σ = electrical conductivity) is very small. This means that the currents induced by the motion are insufficient to change appreciably the applied magnetic field, B.

The ratio of magnetic to inertia forces is

$$S = \frac{D\sigma B^2}{2\rho u} \qquad (1)$$

where ρ is the fluid density. We assume that S is of order one or larger. In this case the ratio of Alfven speed to sphere velocity, $\sqrt{S/Rm}$, is also greater than one. The viscous Reynolds number is large ($Re \gg 1$).

If the Hartman number, $\sqrt{S\, Re}$, is greater than Re, the asymptotic expansion for the flow over the body is known. It is believed that a similar flow occurs when this unequality is not met, provided the Hartman number is large. In any case, there is an upstream wake because $\sqrt{S/Rm} \gg 1$.

For $\sqrt{S\, Re} \gg Re$, the flow over the sphere consists of a column of still fluid extending to infinity upstream and downstream of the sphere (as $\sqrt{S\, Re} \to \infty$). The column is a cylinder whose generator is the equator of the sphere[3,4,5]. If $\sqrt{S\, Re}$ is not greater than Re, it is observed experimentally that the wake upstream has a finite length (longer if S is larger).[7] That is, the still column of fluid has a length which increases as S increases. Such a modification may describe the flow down to S of order one.

The vertical velocity of the sphere is determined by balancing the effects of drag, $\frac{1}{2}\, \rho\, U^2 \frac{\pi D^2}{4}\, C_D$, and buoyancy, giving, for $V/U \ll 1$,

$$U = \sqrt{\frac{2\pi D g}{3\, C_D}\, (1 - \rho s/\rho)} \qquad (2)$$

III. <u>Stability Theory</u>

It is convenient to introduce nondimensional quantities at this point. Non-dimensionalize lengths with $D/2$, velocities with the vertical velocity of the sphere. The pressure is nondimensionalized with ρU^2, and the electric field with UB. The nondimensional laboratory coordinate system is z positive upward, r the cylinder radius, and θ the azimuthal angle. In these coordinates, the flow over the sphere described in the previous section, for S large, takes the form

$$U = \begin{cases} 1, & r < 1 \\ 0, & r > 1 \end{cases} \tag{3}$$

In (I) it is shown that a parallel flow described by (3) is always unstable, however large S. From (I) the nondimensional pressure and the electric field potential have the form

$$p = \text{Real part of} \left\{ P(r)\, e^{i\theta} \quad e^{i\,\alpha\,(z-ct)} \right\} \tag{4}$$

$$\Phi = \text{Real part of} \left\{ \Phi \;\; (r)\, e^{i\theta} e^{i\alpha(z-ct)} \right\} \tag{5}$$

The shape of the mode is a helix as the curves on which r = const, $\alpha z + \theta$ = const, are helixes. From (I), as $\alpha \rightarrow 0$, c becomes

$$c = \frac{1}{2} + \frac{i}{2} \tag{6}$$

From (I), P(r), for $r < 1$ is given by

$$P(r) = AI_1 \;(\alpha r \sqrt{\;1 + \frac{S}{i\alpha(1-c)}}\;), \tag{7}$$

where I_1 is the modified Bessel function of the first kind, of order one. For small α and S at least order one, P(r) becomes

$$P(r) = Ar\sqrt{\frac{\alpha S}{i(1-c)}} \tag{8}$$

From (I), Φ (r), for $r < 1$, is given by

$$\Phi\ (r) = BI_1\left(\alpha r\sqrt{1 + \frac{S}{i\alpha(1-c)}}\right) \tag{9}$$

where B is linearly related to A. For α small, this becomes

$$\Phi(r) = -\frac{iA}{S}\ (1-2c)\ r\sqrt{\frac{\alpha S}{i(1-c)}}\quad . \tag{10}$$

The radial velocity of the disturbance is given by the radial momentum equation:

$$u_r = \text{Real part of}\left\{\frac{1}{S+ i\alpha(1-c)}\left[\frac{-dD}{d\,r}\quad\frac{-iS\,\Phi}{r}\right]e^{i\theta}\ e^{i\alpha(z-ct)}\right\} \tag{11}$$

In view of 8 and 10, this becomes

$$u_r = \text{Real part of}\left\{\frac{2A(c-1)}{S}\sqrt{\frac{\alpha S}{i(1-c)}}\ e^{i\theta}\ e^{i\,\alpha(z-ct)}\right\} \tag{12}$$

For $\alpha \to 0$, the term $\frac{\partial u_z}{\partial z}$ in the continuity equation may be ignored. Hence the perturbation velocity in the θ direction may be found by the continuity equation when the radial velocity is known. This gives

$$u_\theta = \text{Real part of} \left\{ \frac{-i\ 2A\ (c-1)}{S} \sqrt{\frac{\alpha S}{i(1-c)}}\ e^{i\theta}\ e^{i\alpha(z-ct)} \right\} \tag{13}$$

In the next section, equations 6, 8, 12, and 13 are used to relate the force on the sphere to its trajectory.

IV. The Helical Motion

The results of the previous section can be summarized in dimensional form as follows, for $\alpha \to o$:

$$u_r = U\ E \cos\theta \tag{14}$$

$$u_\theta = -\ U\ E \sin\theta \tag{15}$$

$$p = \rho\ U^2\ SE\ \frac{r}{D}\ (-\cos\theta + \sin\theta) \tag{16}$$

The requirement that the sphere move with the horizontal disturbance velocity in the wake, i.e., the wake is attached to the sphere, implies (from 14, 15) that the sphere is moving in the $\theta = 0$ direction with velocity

$$V = UE \tag{17}$$

The force on the sphere due to its horizontal motion is

$$F = \iint p(\bar{n}\ dA) \tag{18}$$

where the integration is over the surface of the sphere.

The force in the $\theta = 0$ direction is (Θ, φ are spherical polar angles)

$$\frac{1}{2} \rho U^2 \left(\frac{D^2}{4} \right) SE \int_0^{2\pi} \int_0^{\pi} \sin^2 \Theta \cos^2 \varphi \sin \Theta d \Theta d \varphi$$

which gives a thrust

$$T = \frac{1}{2} \rho U^2 \left(\frac{\pi D^2}{4}\right) \left(\frac{2S}{3}\right) E \tag{19}$$

Likewise, there is a lift force in the $\theta = \frac{3\pi}{2}$ direction of

$$L = \frac{1}{2} \rho U^2 \left(\frac{\pi D^2}{4}\right) \left(\frac{2S}{3} \right) E \tag{20}$$

Provided the radial acceleration is sufficiently small, (i.e. of R is the radius of the helical path of the sphere, $D/R << 1$), the forces on the sphere as it moves in a helical path are given by 22 and 23.

To have a steady motion, the thrust must equal the horizontal component of drag. For $V/U << 1$, this is

$$T = \frac{1}{2} \rho U^2 \left(\frac{\pi D^2}{4}\right) C_D (V/U)$$

which can be written as

$$C_D = \frac{2}{3} S \tag{21}$$

Let the apparent mass of the sphere in the horizontal direction be

$$K \rho \frac{\pi D^3}{6} \tag{22}$$

For steady motion, the mass of the sphere (density ρ_s) plus the apparent mass times the radial acceleration must equal the lift:

$$L = \frac{V^2}{R} \left(\rho_s + K\rho\right) \frac{\pi D^3}{6} ,$$

which can be written as

$$R = \frac{2ED}{S} \left(\frac{\rho_s}{\rho} + K\right) \tag{23}$$

The wave length, Λ, for the helical path of the sphere is the time for one revolution times the vertical velocity. From 26,

$$\Lambda = \frac{4\pi D}{S} \left(\frac{\rho_s}{\rho} + K\right) \tag{24}$$

Equations 21 and 24 are the principal results of this paper. Equation 21 forms the basis for a possible experimental test of the theory.

Acknowledgements

The author wishes to thank the staff of the Ionosphere Research Laboratory who aided in preparation of this manuscript. This work was supported by NASA Grant NsG-134-61.

Footnotes

1. Liepmann, H. W., Hoult, D. P. and Ahlstrom, H. G., 1962 Miszellen der Angewandten Mechanck-Akademie Verlag-Berlen p. 175.

2. Hoult, D. P.,"The Effect of An Axial Magnetic Field on the Stability of an Axisginmetric Jet or Wake" Submitted to Phys. Fluids, Nov. 1964.

3. Stewartson, K.,1960 Journal of Fluid Mechanics 8, 82.

4. Chester, W.,1961 Journal of Fluid Mechanics 10, 459.

5. Childress, S.,1963, Journal of Fluid Mechanics 15, 437.

6. Batchelor, G. K. and Gill, A. E., 1962, Journal of Fluid Mech. 14, 529.

7. Ahlstrom, H. G., 1963, Journal of Fluid Mechanics 15, 205.